

## Methodology for an integrated modelling of macro and microscopic processes in urban transport demand

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### Abstract

The paper presents the theoretical formulation and the underlying assumptions for an activity-based approach of transport demand modelling. Starting with the definition of a time hierarchy of decision-making in the urban environment, rules are formulated that dictate the general hierarchic structure of individuals' choices in the urban system. The temporal scale defines decisions for activities and their daily sequence, the geographical scale decisions associated to destination choice processes. We build activity plans (number and daily sequence of activities) from an empirical data set and calculate trip paths (time-spatial trajectories including transport modes and travel destinations) assuming consumers to maximize their utility in the decision-making process. First results of the translation of the theoretical model into a real-world application are shown for the city of Santiago, Chile.

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## **1. Introduction and Motivation**

The development of appropriate methodologies for modelling urban transport processes is of paramount importance since they are utilized as planning instruments that support decision-making. Models are applied within planning units and they serve for the analysis of effects of infrastructure or transport demand management measures and help to monitor the performance of transportation systems in major cities. To this purpose, models have to deal with complex urban environments and processes on different spatial and temporal scales. In this sense, one would expect that urban transport models are able to handle different challenges: be responsive to changes that influence transport demand on a long-term, like new job opportunities or changes in land-use and to be sensitive to adjustments on the short-term, like daily activity-scheduling in response to e.g. new shopping or leisure opportunities.

Some of the well-known strategic urban transport equilibriums models (such as the 4-step models ESTRAUS, EMME-3 or VISUM), highly simplify the complexity of urban systems in order to represent reality in a single scale for each of the most relevant dimensions: time, space and population clusters. At the same time, they are trip-based and often consider a small number of travel purposes, distinguish not more than three, usually one or two time periods and divide the demand side in a limited number of user classes. Today these models are popular to support strategic decision-making concerning environmental and fare policies as well as projects of road and public transport infrastructure. Despite the simplifying assumptions mentioned, their pay-off is that equilibrium conditions can be imposed, which yields analytical models, accepted level of accuracy on estimates of congestion levels and the framework is consistent with microeconomic theory.

A direction to develop more detailed models in urban land-use has been to develop system simulations (Waddell, 2002; Hunt and Abraham, 2007), which by eliminating equilibrium conditions reduce complexity with the benefit of an increase in modelling details. The gains are particularly on the spatial representation of the system as well as in the disaggregation of agents, maintaining the single scales in geography and time. Further disaggregation of time scales and users, has been introduced in a family of models based on the concept of activity-based analysis (Bhat et al., 2008; Bowman and Ben-Akiva, 2000; Rilett, 2001; Justen and Cyganski, 2008), which understand travel demand as derived from interdependent activities and trips chains throughout a day. Their main contribution is the increased detail on the chain of activities and

destinations and the interdependency among them. Again, the more realism provided by this models is at the cost of departing from the equilibrium paradigm.

Thus, recent models have increased the spatial detail (urban simulators) and the time dependence on activities (activity based) (for an overview of transportation and land-use models under development and in application see Davidson, 2007; Iacono et. al, 2008; Buliung and Kanaroglou, 2007). A common problem of these models is the difficulty on handling the combinatorial number of choices and the dependency among them that arise from activities, spatial location, travel options and scheduling activities for all consumers.

In this paper, we develop a methodology and show first empirical results that aim at introducing further details in the analysis of the location and travel demand model concerning users, space, time, activities (trip purposes) and travel modes, as well as on sequencing activities and trips. Our approach is to design a multi-scale model of the urban system, applying classical concepts of hierarchical processes drawn from the literature of dynamic systems that were originally observed in ecological systems and subsequently extended to economic and social systems (Gunderson and Holling, 2002). In all these systems, the complex combinatorial process as a result of the huge amount of possible individual choices is simplified by means of a hierarchical approach, which describes a natural order of systems dynamics.

We particularly concentrate on time and spatial scales, with the nomenclature of bi-scales (called micro and macro scales) as an implementation of the approach describing the activities and trips performed in a single day by a heterogeneous population. We first define a notation for activity and trips chains; then we specify a theoretical and hierarchical decision model based on the microeconomic theory of consumers behaviour which we simulate in a theoretical context to compare the effect of the hierarchical approach compared with the classical static equilibrium model. The macro-micro approach is currently under implementation for the case of Santiago City, combining information provided by the existing macro scale models of land-use and transport (MUSSA and ESTRAUS) with detailed information of consumers' choices obtained from Santiago's Travel and Household Survey (EOD).

## 2. The Hierarchical Model

### 2.1. The hierarchical structure of choices

A basic assumption of the methodology is that choices for activities and later on for related destinations and travel modes are modeled as they were made hierarchically. Individuals first choose an activity plan referring to the set of activities and their order and timing, after that they choose a trip path, including the location of activities and the transport mode. The daily activity plan, including its associated trip path is called the individual's program. The individual is assumed to perform a search and evaluation process for an optimal program. This process is extremely complex for human beings due to the huge amount of options that defines the feasible set: the combination of ordered activities, locations, transport modes and the allocation of time. Such complexity means also high requirements for advanced computers to simulate such a process. The number of options increases with the number of ways in which time can be allocated and the set and order of activity plans performed. On the spatial dimension, the number of options increases with the square of the number of location points that describe origin-destinations alternatives; thus, a crucial parameter in the magnitude of the combinatorial set of options is the level of detail that describes the geography, i.e. the zoning system. Additionally, the individual search needs to consider the limited amount of resources available. In order to introduce a rational strategy to reduce complexity, the information can be organized into a hierarchical structure of space and time. This structure, common to several complex systems both in nature and in social organizations, is a plausible strategy as long as we assume that alternatives from different scales are substantially different in their contribution to happiness and/or on their consumption of resources that constrain the consumer's rational choice domain.

Searching and choosing on the set of options is simplified under a spatial hierarchical strategy. First, searching takes place on the macro level, i.e. among a small number of zones, second, the search is extended to a further disaggregated space, a micro spatial level, but at every time limited to the area defined by the chosen macro zones. The macro level is spatially characterized by macro zones, comprising around 600 Traffic Analysis Zones (TAZ) in Santiago, while the micro level is a disaggregated zone system which in Santiago comprises approximately 50,000 city blocks. In this context a direct search on the city block micro level implies 2.500 million points, while a hierarchical search reduces this number to a maximum of 360.000 macro zones plus, on average, 2,500 micro zone points.

The structure is also adequate to model the differentiated dynamic of subsystems at each temporal scale: at a micro level the speed of change of variables is higher (small time windows) than those at a macro scale, while changes at the macro scale have dominant impacts on the micro scale. For example, a secondary activity, such as daily shopping, takes place in a time window of the order of one hour and choices of their location usually occur in the vicinity and dependent on the location of a primary activity conducted before (like home or work location). Conversely, the work activity generally consumes a large part of the time budget and changes in job locations usually happen in a time window of years in the context of the whole city. An example of the time-hierarchy in a city is shown in the following Table 1, which of course contains assignments of activities to cells subject to analysis in specific contexts.

**Table 1: Time hierarchy for activities**

TIME-WINDOWS	TIME-UNITS	CONSUMERS	SUPPLIERS	REGULATORS
<b>Very long-term</b>	Centuries	Cultural factors	Cities building and network structure New cities	
<b>Long-term</b>	Decades to Years	Family structure Education Car-Ownership Residence Job	<u>Infrastructure:</u> Buildings Roads/Tracks Bridges Technology	<u>Plans:</u> Regulations Incentive Policies Infrastructure Plans
<b>Short to very short-term</b>	Years to Months	Time and location of leisure activities Transport Modes	<u>Operations:</u> Level-of-Service	
<b>Very short-term</b>	Days to Hours	Route Choice Local Transport Modes Walking Destinations	<u>Adjustment of Operations:</u> Stops and Delays	

Each decision has to be made on the appropriate scale for the individual's choice, for what we consider the following rules:

1. *Hierarchies of scales:* the geographical scale is subordinated to the temporal scale.
2. *The resources structure:* The scale of each process or activity choice is directly associated with the amount of resources required.

3. *Self-contained subsystems*: Every process (constraint) that exceeds the dimensions of the micro scale has to take place and be defined at the macro level. Conversely, every process (constraint) that is fully embedded in a micro scale has to be described at the micro scale.
4. *Macro–Micro temporal dependency*: The dynamic of a micro scale system is constrained by the slow moving variables at the macro level. Conversely, micro level fast moving variables influence future macro variables.

This set of axiomatic rules dictates the general hierarchical structure of individuals' choices in urban systems having the following implications in building hierarchical models: The temporal scale defines the long-term effects of macro scale choices, both in terms of the consumption of resources and the level of utility attained. The appropriate geographical scale is one that provides efficiency for a complex search processes, saving efforts and resources, but does not affect fundamentally the long-term quality of life. Therefore, the rule that subordinates the geography scale to the temporal scale implies that the formed may be univocally defined by the latter.

Macro level processes consume and produce a larger amount of the individual scarce resources than micro level ones. Resources consumption - time and wealth - are defined at the macro level including the savings to be spent at the micro level. Hence, it is more profitable for the consumer to invest time on searching in a larger spatial context - the macro scale - than identifying detailed differences at a micro scale. The amount of information required to make informed choices at each sub-system is bounded to the subsystem hierarchy, which is consistent with the existence of a maximum amount of feasible effort made by human beings in a search process, be that physically or economically defined. The implication is that by assuming consumers' efficient hierarchical strategies we reproduce the real behavior and save computing resources. Saving computing resources becomes crucial when demand model enter equilibrium processes where demand calculations increase exponentially.

## 2.2. The hierarchical theoretical model: Notation

The notation is defined in order to provide complete flexibility to specify the activity-travel demand model in multiple scales. The temporal scale defines primary, secondary and other levels of activities while the geographical scale defines the levels relating to the spatial context, for example macro and micro locations.

$k$  : Index denoting the temporal scale for activities,  $k=1, \dots, K$ ; we use the convention that  $k=1$  is the largest time window and  $k=K$  the smallest time window. In the bi-

scales model,  $k=1$  is the macro scale and  $k=2$  is the micro scale;  $K=2$ . We also assume that time windows are well defined, i.e. a macro time window at level  $k$  is disaggregated into an integer number of micro time windows at level  $k'=k+1$ , hence a micro time window does not overlap over two macro time windows.

$g$  : Index denoting the geographical or spatial scale,  $g=1, \dots, G$ ; we apply the convention that  $g < g'$  implies geography  $g$  is more aggregate than  $g'$ . In the bi-scales model,  $g=1$  is the macro scale and  $g=2$  is the micro scale in the geographical dimension;  $G=2$ . We assume that the geography is also well defined, i.e. a macro zone at level  $g$  is disaggregated into an integer number of micro zones at level  $g+1$ , hence no micro zone overlaps over two macro zones.

$H$  : Set of individuals' socio-economic classes.

$Z^g$  : Set of location zones defined at the geographical level  $g$ . The map  $M(z \in Z^g, z \in Z^{g+1})$  defines the partition of each zone at geographical level  $g$  into the level  $g+1$ ; we denote  $\tilde{Z}^{g+1/g} = \{z_j^{g+1} \in Z^{g+1} / M(z \in Z^g, z \in Z^{g+1})\}$ . Under map  $M$  every micro zone is completely contained in a macro zone.

$A^k$  : Set of activities associated at the temporal level  $k$ , with  $A^k = \bigcup_{k' < k} \tilde{A}^{k'}$ , with  $\tilde{A}^k$  the set of activities decided at the temporal window  $k$ . Thus,  $A^k$  is the aggregated set of activities decided at temporal scale  $k$  and at all temporal scales  $k' \in (1, \dots, k-1)$ , then  $A^k \subset A^{k+1}$ .

$n = (h, i), h \in H, i \in Z^g$  : Index denoting an individual, identified by socio-economics  $h$  and located at zone  $i$ .

*Activity plans*:  $\zeta_n^k = \{a_{e/n}^k; a \in A^k; e = 1, \dots, e_n^k\}$  is the tuple denoting the  $n^{\text{th}}$  individual's daily sequence of activities, described by the sub-set of activities at temporal scale  $k$  scheduled in the temporal sequence defined by index  $e$ ;  $e_s$  is the number of daily activities.

Example: Consider the following activity sets:  $\tilde{A}^1 = (H, W)$  and  $\tilde{A}^2 = (S)$ , with  $H$  for home and  $W$  for work the set of primary activities ( $k=1$ ) and  $S$  for shopping the secondary activity ( $k=2$ ). An individual  $n$ 's activity plan is  $\zeta_n^{k=2} = (H, W, S, H)$ , with four activities, i.e.  $e_n^{k=2} = 4$ ; this is represented at the macro temporal scale as  $\zeta_n^{k=1} = (H, W, H)$ , with three activities, i.e.  $e_n^{k=1} = 3$ . It follows that  $e_n^k < e_n^{k+1}$ .

This notation allows us to combine the hierarchical structure of activities in the temporal scale with the sequential representation of the activity plan at any given temporal scale. It is particularly relevant to emphasize that the engagement in each activity is associated to the appropriate time window and remains unchanged during that time window, although activity plans include activities of the reference temporal scale ( $k$ )

and of all other higher levels ( $k' < k$ ). This is despite the fact that all these activities are performed within the same time period  $\tau_0$  (say a day).

*Trip paths:*  $v_{\zeta_n}^g = \{v_{e/\zeta_n}^g; e = 1, \dots, e_n^k\}$  is the tuplex defining the *trip stages* in the trip path.

There is one stage per activity visited, ordered in the sequence of the activity plan. Trip paths can be described at any geographical scale for trips. Each trip stage  $v_{e/\zeta_n}^g = (l_{q/e, \zeta_n}^g; q = 1, \dots, q_g)$  is a set of triplexes identifying the *trip links* (each one describing the origin and destination zones denoted by  $d_{q/e-1}$  and  $d_{q/e}$  respectively) and mode of transport denoted by  $\omega_{q/e}$ ; then  $l_{q/e, \zeta_n}^g = (d_{q/e-1}^g, d_{q/e}^g, \omega_{q/e}^g; d \in Z^g, \omega \in \Omega^g)_{e, \zeta_n}$ .

The number of links is  $q_g = 2g - 1$ . To explain this, consider a geography represented in a single spatial level ( $g=1$ ), then a trip stage contains only one link, which is the minimum number of links. Consider now a given trip stage represented in a hierarchical spatial structure with  $g=2$ , called macro and micro zones as shown in Figure 1. It represents the trip from the micro zone  $i$  of macro zone  $I$ , to the micro zone  $j$  of the macro zone  $J$ , using the following sequence of transport modes:  $m^-$  (departure local mode),  $M$  (long distance mode) and  $m^+$  (arrival local mode). This trip stage is denoted with  $v_{e/\zeta_n}^{g=2} = ((i, I, m^-), (I, M, J), (J, j, m^+))$  for the micro scale and  $v_{e/\zeta_n}^{g=1} = ((I, M, J))$  for the macro scale.

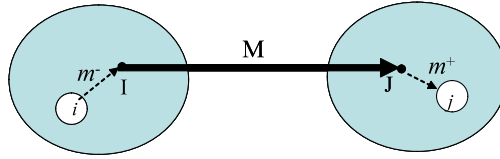


Figure 1: The micro-macro-micro trip link

Under this notation, in a trip path defined geography level  $g$  we represent the destination zone at all geography levels  $g$ . At  $g'=g$  the destination zone of the trip stage  $e$  is  $d_{q_g/e}^g$  (represented by the micro zone  $d_{q=3/e}^g = j$  in the example), and the same destination is represented at the next aggregated geography level ( $g'=g-1$ ) by the destination zone  $d_{q_{g-1}/e}^g$  (the macro zone  $d_{q=2/e}^g = J$  in the example).

### 2.3. The hierarchical utility maximization problem

The consumer's behaviour is modeled assuming the classical utility maximization paradigm. Following De Serpa (1971) and followers, utility is assumed to be yield by the use of the time budget and wealth expenditure on goods and services. We expand this perspective as we innovate through the specification of quality of the activity performed as differentiator of the utility level attained by the consumer. Time includes its allocation to the activity ( $t_e$ ) and travel duration ( $tv_e$ ) at each stage  $e$ . The quality is assumed to depend on the environment at the activity location  $d_e = d_{q_g/e}^g$ , described by



the vector of land use  $l_e = l(d_e)$ , and the level of consumption assigned to the activity, denoted by vector  $x_e$ .

This means that a time schedule program which includes an activity plan and a trip path must be defined at a given temporal scale, denoted  $T^k$ . This is associated with the following decisions: what activities to perform ( $\zeta_n^k$ ), their duration ( $t_e$ ), sequence ( $e$ ), and location ( $l_e$ ), how much goods and services to consume ( $x_e$ ), and how much time spent on traveling ( $tv_e$ ). The utility derived from a program by individual  $n$  is:  $U_n(\Gamma^k) = U_n(t_e, tv_e, l_e^g, x_e; \forall e = 1, \dots, e_n^k)_{\zeta_n^k, v^g}$ .

The utility maximization problem must allocate time and monetary resources constrained to the feasible domain regarding the set of resources wealth ( $S$ ) and time ( $\tau$ ). At each temporal level  $k$  the feasible domain is defined by net wealth and time made available from longer term choices, and what is spent and saved for smaller term time window decisions. Additionally, following De Serpa we assume that associated to the consumption of goods, a minimum of time is required. We specify the following domain:

$$\begin{aligned} \sum_{a^k \in \zeta_n^k} y_n^a - p \cdot x_n^a + S_n^k &= S_n^{k-1}, \quad \forall k = 1, \dots, K \\ \sum_{a^k \in \zeta_n^k} t_n^a + tv_n^a + \tau_n^k &= \tau_n^{k-1}, \quad \forall k = 1, \dots, K \\ t_n^a &= f(x_n^a) \quad \forall a \in \zeta_n^k, k = 1, \dots, K \end{aligned} \quad (1)$$

where each activity is assumed to potentially yield income ( $y^a$ ) and induce expenditure depending on consumption and the prices  $p$ . At any time window  $k$ , the income disposable is  $S^{k-1}$  and the time disposable is  $\tau^{k-1}$ . Here resources are defined by unit of time period (say 24 hours). At the beginning of any long term decision period (i.e.  $k=1$ ) each individual has an initial wealth  $S_n^0$  and all individuals have the same time period budget  $\tau^0$  (24 hours). At the most shorter term  $k=K$ , we define that  $S_n^K = 0$  and  $\tau_n^K = 0$ , which ensures that all resources are consumed.

In a multi-scales model with temporal and spatial hierarchies the utility optimization problem that yields the individuals rational program choice may be specified in several ways, depending on a spatial and temporal hierarchical structure.

*Assumption 1: The geographical scale is subordinated to the temporal scale, denoted by  $g(k)$ .*

This means the allocation of a higher order in the consumer's behaviour to the temporal scale under the argument that it is on this scale where the consumer assigns the relevant resources to activities (e.g. income and time). This does not ignore that the spatial scale influences the consumer's behaviour as well since the individual's cognitive map of a space is hierarchical in nature and represents an individual's

selective search strategy in a complex space. Contrarily, decisions on the temporal scale involve a lump amount of resources whose magnitude and time window for the next adjustment is exogenous to the individual. Thus, the temporal scale is substantial to the individual's behaviour, while the spatial representation is an auxiliary tool.

*Assumption 2: The optimal program specified in a temporal scale  $K$ , can be decomposed into a hierarchical structure of conditional optimal sub-programs  $k \leq K$ , each one conditional on the optimal sub-programs  $k' < k$  and parametric on the set of the individual's total resources.*

The rationale of Assumption 2 is that every program at temporal scale  $k$  can only adjust activities associated to that level while activities decided at levels  $k' < k$  are fixed and impose a set condition. Additionally, at level  $k$  the individual has to save resources to make choices on micro levels  $k' > k$ , hence decisions are also conditional on the savings.

Under Assumptions 1 and 2 we propose the following optimization problem, called  $P_K$  for a temporal scale span specified in  $K$  levels.

$$P_K) \quad \left[ \begin{array}{c} \text{Max}_{a^{k=1} \in A^{k=1}} \left[ \begin{array}{c} \text{Max}_{l \in Z^g(k)} U_n(\Gamma^{k=1}) \\ \omega \in \Omega^g(k) \\ t_a \in [0, T] \\ x_a \in R(S^k) \end{array} \right] \quad \dots \quad \text{Max}_{a^k \in A^k} \left[ \begin{array}{c} \text{Max}_{l \in Z^g(k)} U_n(\Gamma^k) \\ \omega \in \Omega^g(k) \\ t_a \in [0, \tau^k] \\ x_a \in R(S^k) \end{array} \right] \quad \dots \end{array} \right]_{\zeta_n^{k-1}, v_{\zeta_n^{k-1}}^{g(k-1)}} \dots$$

$$\left[ \begin{array}{c} \text{Max}_{a^K \in A^K} \left[ \begin{array}{c} \text{Max}_{l \in Z^g(K)} U_n(\Gamma^K) \\ \omega \in \Omega^g(K) \\ t_a \in [0, \tau^K] \\ x_a \in R(S^K) \end{array} \right] \end{array} \right]_{\zeta_n^{K-1}, v_{\zeta_n^{K-1}}^{g(K-1)}}$$

The solution of problem  $P_K$  should be in the domain defined by equations (1) which is imposed by the domain of time and consumption variables in the maximization process. This hierarchical optimization problem assumes that individuals first decide the long term activities of an activity plan and the trip path, then he/she decides the next level (shorter term) of activities-trips choices conditional on the longer term activity-trip choices and so on. It follows that at each temporal window  $k$  the optimization process is conditional on the solution of all longer temporal scales  $k' < k$ . This sequential optimization process is linked by the allocation of the common resources of time and income.

For the bi-levels spatial-temporal model ( $K=2$ ) the optimal program is the solution of:

$$P_2) \left[ \begin{array}{l} \begin{array}{l} \underset{a^1 \in A^1}{Max} \left[ \underset{\substack{z \in Z^1 \\ \omega \in \Omega^1 \\ t, x}}{Max U_n(\Gamma^1)} \right] \\ s.t. \sum_{a^1 \in \zeta_n^1} y_n^a - p \cdot x_n^a + S_n^1 = S_n^0 \\ \sum_{a^1 \in \zeta_n^1} t_n^a + tv_n^a + \tau_n^1 = \tau^0 \\ t_n^a = f(x_n^a) \quad \forall a \in \zeta_n^1 \end{array} \\ \begin{array}{l} \underset{a^2 \in A^2}{Max} \left[ \underset{\substack{z \in Z^2 \\ \omega \in \Omega^2 \\ t, x}}{Max U_n(\Gamma^2)} \right]_{\zeta_n^1, v_{\zeta_n^1}^1} \\ \sum_{a^2 \in \zeta_n^2} y_n^a - p \cdot x_n^a = S_n^1 \\ \sum_{a^2 \in \zeta_n^2} t_n^a + tv_n^a = \tau^1 \\ t_n^a = f(x_n^a) \quad \forall a \in \zeta_n^2 \end{array} \end{array} \right]$$

with  $k=1$  ( $g=1$ ) a macro temporal (spatial) level and  $k=2$  ( $g=2$ ) a micro temporal (geographical) level .

Observations:

1. *Time coordination.* The above multi-scales model considers that all activities decided at a given time level  $k$  are decided instantaneously at the beginning of the time window  $k$ , and remain constant along the time window. In this context, when the individual optimizes choices at the end of a time window of level  $k$ , all faster choices made at  $k' > k$  can be adjusted simultaneously, hence the optimization problem at level  $k$  is only conditional on slower variables decided at  $k' < k$ . It follows that the utility supreme of the activities programs is attained at the end of a time window of level  $k=1$ , when all variables are adjusted simultaneously.
2. *The indirect utility:* At any time level, the optimization problem is conditional on the budgets of time and wealth and on the program (activities and path) decided at all long term temporal levels. Hence the indirect utility is denoted by  $V_n^{k_0} = V_n(\Gamma^{k_0}; \Gamma^{k-1}, S^k, \tau^k; \forall e=1, \dots, e_n^k)$ , with  $\Gamma^k = (t_e^k, tv_e^k, l_e^{g(k)}, x_e^k; \forall e)$  the set of decision variables at each time window  $k$ . Notice that function  $V_n^k$  is recursive on macro level programs leading to the direct consequence that current behaviour on short term choices is conditional on all longer term choices. Another consequence is that individuals which are equal in all socio-economic characteristics may be observed choosing different sets of activity-trips because their set of choices made in the longer term activities are different. This conditional choice process also includes durable goods, such as car ownership or housing, which affect short term decisions in the transport system: the choices for travel modes, routes and locations of short term activities.

3. *Resource dependency.* We remark that the hierarchical optimization problem at time scale  $k$  and the corresponding indirect utility function are explicitly dependent on the set of resources saved for activities to be decided at  $k' > k$ . Indeed, observe that at  $k=1$  in  $P_2$  the optimum choice is to exhaust all resources making  $S^1$  and  $\tau^1$  equal to zero which is not a reasonable outcome because the arbitrary time scale chosen for a given type of analysis define the way in which resources are allocated. Thus, dependency is both ways, with times scales of longer and smaller terms.
4. *Memory.* The above framework with short term choices being dependent on all long term choices introduces a limited form of memory in the decision process. Indeed, decisions made at any temporal level take into account the decisions at all longer term levels but this does not represent the memory on the history of decisions. In fact, at the end of time level  $k=1$  all decisions are optimized simultaneously without memory. The introduction of full memory can be introduced as an extension of this model.

## 2.4. The random utility multi scales approach

In order to develop a decision model that considers idiosyncratic variability we consider that consumers face shocks on their decisions. Shocks are assumed to be temporally and spatially dependent on the associated scales, such that long-term choices are subject to shocks at the macro scale affecting the level of resources (time and income) in the order of magnitude of the associated scale.

We propose the following hierarchical random utility optimization problem considering a time scale span  $K$  (denoted by  $RUP_K$ ):

$$\begin{aligned}
 RUP_K) \quad & \left[ \begin{array}{c} \text{Max}_{a^{k=1} \in A^{k=1}} \left[ \begin{array}{c} \text{Max}_{z \in Z^{g(k)}} U_n(\Gamma^{k=1}) + \varepsilon^{k=1} \\ \omega \in \Omega^{g(k)} \end{array} \right] \quad \dots \quad \text{Max}_{a^k \in A^k} \left[ \begin{array}{c} \text{Max}_{z \in Z^{g(k)}} U_n(\Gamma^k) + \varepsilon^k \\ \omega \in \Omega^{g(k)} \end{array} \right] \end{array} \right]_{\zeta_n^{k-1}, v_n^{g(k-1)}} \\
 & \dots \quad \text{Max}_{a^K \in A^K} \left[ \begin{array}{c} \text{Max}_{z \in Z^{g(K)}} U_n(\Gamma^K) + \varepsilon^K \\ \omega \in \Omega^{g(K)} \end{array} \right]_{\zeta_n^{K-1}, v_n^{g(K-1)}} \\
 & \sum_{a^k \in \zeta_n^k} y_n^a - p \cdot x_n^a + S_n^k = S_n^{k-1}, \quad \forall k = 1, \dots, K \\
 & \sum_{a^k \in \zeta_n^k} t_n^a + tv_n^a + \tau_n^k = \tau_n^{k-1}, \quad \forall k = 1, \dots, K \\
 & t_n^a = f(x_n^a) \quad \forall a \in \zeta_n^k, k = 1, \dots, K
 \end{aligned}$$

Previous to the analysis of this problem we consider the following example: the choice for the activity shopping can be subject to shocks, like weather conditions that disrupt the daily schedule affecting shopping time and the location choice. We may observe that the effect of this shock vanishes in the time window of a week because rescheduling takes place and expenditure is readjusted. It seems also reasonable that a shorter term shock like a minute delay in a congested junction does not alter the daily scheduling since in the next hour or so such a delay vanishes by readjustments of times. A longer term shock like a sudden acceptance in a new job alters all shorter term decisions, including shopping, because the new job implies a specific commitment to time and a different income. This example shows that shocks at time scale  $k$  affects all decisions at scales  $k' > k$ ; conversely, it does not affect decisions at scale  $k' < k$ .

The above argument implies the following:  $Cov(\varepsilon^k, \varepsilon^{k'}) \begin{cases} = 0 & \text{if } k > k' \\ \neq 0 & \text{if } k \leq k' \end{cases}$ , then the covariance matrix is assumed to be triangular superior. This implies that the solution of the problem is a joint probability of the following form:

$$P_n(\Gamma^K) = P_n(\Gamma^1) \cdot P_n(\Gamma^2)_{\Gamma^1} \cdot \dots \cdot P_n(\Gamma^K)_{\Gamma^{K-1}} \quad (2)$$

where each choice probability is independent on the choice probabilities of activities decided at smaller time scales ( $k' > k$ ), but conditional on the choice probabilities of activities decided at larger time scales ( $k' < k$ ). Notice that (2) implies the following decomposition or multiplicative property:  $P_n(\Gamma^K) = P_n(\Gamma^{K-1}) \cdot P_n(\Gamma^K)_{\Gamma^{K-1}}$ .

This property is valid for any time scale  $k < K$  under the condition that the allocation of total resources is consistent with the activities to be decided at all time scales. In other words, a program  $\Gamma^{k < K}$  must be a subprogram of a full time scales program  $\Gamma^K$  because the latter provides the feasibility conditions on resources at all scales. Notice that this condition links all decisions associated to a scale  $k' > k$  in a way that imposes that all these decisions should be taken simultaneously. However, in the dynamic choice process described above these is feasible and necessary. To understand why, notice that at the end of the time window in scale  $k$ , all time windows at scales  $k' < k$  also elapse because fast moving variables are readjusted any time a slower moving variable is modified. Therefore, we conclude that choices at a given time window are conditional on the resources available from longer term choices and are decided simultaneously with decisions made at smaller time scales.

## 2.5. Dependency on the geographical scale

Within a temporal scale  $k$ , the problem  $RUP_k$  includes location decisions for all activities decided at that scale (conditional on activities and locations decided at longer term scales). Thus, let us consider the following example: the individual is searching an optimal location for shopping. From time scales  $k-1$  he/she has a limited amount of time and income, say three hours and 50 dollars which limits the maximum return travel time to say 2 hours (one hour on each direction). This limit defines a suitable geography scale  $g$  such that: i) the time spent in mental searching is reasonable compared to the travel and activity duration time (say a couple of minutes); ii) it fits with the limited

information handled by the consumer. For a one hour travel radius the individual may search at two levels, the macro zone TAZ-level for a subset of most attractive city sub-centers and then at a micro zone block level for a specific store option. Shocks may affect the location choice at the TAZ level, for example because the car has few gasoline reducing the maximum distance, and at a block level for example because congestion in a given street has blocked access to park nearby the store.

Let us define the marginal probability of performing a given activity-trip at stage  $e$  as  $P_n(e \in \Gamma^k) = P_n(t_e^k, tv_e^k, l_e^{g(k)}, x_e^k)_{\Gamma^{k-1}}$ . Assuming that the activity, say shopping, is decided prior to other travel choices, then we decompose  $P_n(e \in \Gamma^k) = P_n^a(t_e^k, x_e^k)_{\Gamma^{k-1}} \cdot P_n^t(tv_e^k(\omega, l), l_e^{g(k)})_{\Gamma^{k-1}, t_e, x_e}$  where the travel time is dependent on the mode and location choice. Notice that this decomposition imposes that the activity choice marginal probability ( $P^a$ ) and the corresponding trip choice as a conditional probability ( $P^t$ ) on the activity choice are jointly feasible. For example, one will not choose to go for shopping if the closest location option is beyond the time and budget available at the time scale of shopping. Secondly, notice that  $P^t$  is conditional on  $P^a$ , then on  $t_e$  and  $x_e$ . These two considerations make the decomposition assumption plausible though not sufficient because it is possible to identify activities where activity and trip choices are taken jointly. Nevertheless, since the trip choice probability concentrates and isolates the geographical choice it is useful to analyze the choice process in the spatial scale. Similarly to time shocks, in the shopping example above we observe that in the geographical scale shocks at scale  $g$  do not affect spatially wider decisions at  $g' < g$ , and the decisions at level  $g$  are conditional on shocks at  $g' < g$ . This implies the following spatial multiplicative property:

$$P_n^t(\omega, l)^g = P_n^t(\omega, l)^{g-1} \cdot P_n^t(\omega, l^g)_{(\omega, l)^{g-1}}$$

$$\text{with } P_n^t(\omega, l)^g = P_n^t(tv_e^k(\omega, l), l_e^{g(k)})_{\Gamma^{k-1}, t_e, x_e}.$$

We emphasize that, although the multiplicative property applies to both the time and geographical scales it is valid in a more general context for the time scale.

## 2.6. Conclusions on theoretical construct

The above hierarchical approach provides the base structure to develop a family of urban models, each of them specific on the set of decisions, the spatial-temporal scales and the way shocks are modeled. The multi scales approach is modular because it permits that each of these models represent a sub-problem of a larger one, which can be modeled by extending the model by allowing more scale levels.

In section 3 next, we show some simulation tests built on a hypothetical case of a single agent who pursues the maximization of his utility (deterministic choice) based on geographical and temporal decisions associated with work and leisure (shopping). The objective of the experiments is to visualize the sub-optimality coming from constraints on long-term decisions when the individual is facing shocks on the market of goods through changes in prices, comparing the outcome of a single stage model with the

multiple-time-scale model which represents a hierarchical process in the individual's decision making considering the possible flexibilities and limitations at the different levels (scales).

Then, in section 4 we present an application of the model to the case of activity-based travel demand limited to only two spatial and temporal scales, called the bi-scale mode. In its empirical implementation probabilities are calculated from data instead from a functional probability. The aim is to assess the value of the hierarchical approach and the potential benefits before developing an implementation of a fully operational model.

### 3. Simulation experiments

#### 3.1. Simulation framework

Let us assume a single individual with home location fixed at point 0. In terms of geography, to simplify the analysis in these experiments we consider only one physical dimension, which is a straight line starting at the fixed home location at 0, and extended until a boundary at a distance  $L$  from the origin home. On that line, the individual decides both, the work and the shopping locations, the former to earn money and the latter where s/he can buy the goods required to realize leisure activities either at the shopping place or at home. In these experiments, the individual does not save money, which means that all income is spent in buying goods.

In addition, the individual can decide how many hours to work and the leisure time, assuming a fixed time required to sleep. We assume that between work, travel and leisure (including shopping) s/he has 16 hours available. The individual can work at any place along the geographical line; however, wage rates are differentiated depending on the job location. In addition, s/he can buy goods anywhere along the line, although the price changes depending on the shopping location.

The wage ( $w$ ) rate of the place for work ( $w(l_w)$ ) is then represented by a Gaussian function and the price of a bulk shopping is represented by an inverted Gaussian function. Analytically,

$$w(l_w) = w_b + w_f \cdot \exp\left(-\frac{\left(\frac{l_w - \ell_w^{\max}}{L}\right)^2}{2w_c^2}\right) \quad (3)$$

$$p(l_{sh}) = p_{\min} + p_c - p_f \cdot \exp\left(\frac{-(\ell_{sh} - \ell_{sh}^{\min})^2}{2p_c^2}\right)$$

In the above expressions  $\ell_w, \ell_{sh}$  represent choices of work and shopping locations respectively.  $\ell_w^{\max}, \ell_{sh}^{\min}$  are the locations of the maximum salary and minimum goods price respectively (the most attractive locations for developing both activities).

$w_b, w_c, w_f, p_c, p_f$  are the parameters of the Gaussian functions, which define the shapes and dispersion of the curves. The value of  $p_{\min}$  can be interpreted as the actual minimum price happening at the optimal location for shopping  $\ell_{sh}^{\min}$ . In the case of the wage rate, we assume a minimum wage of  $w_b$  summed to the Gaussian curve. Notice that for computational issues, the Gaussian for the wage rate was normalized.

With regard to the availability of transport modes, the individual has access to two modes, private car and bus. In the case of car, s/he pays an operational cost in (\$/hour), so s/he spends money depending on the traveling time. In the case of bus, the individual pays a fixed fare per trip. In total, independent of the chain of trip decided by the individual, s/he will perform three trips always, either H-W-Sh-H or H-Sh-W-H (with H, W and Sh are home, work and Sh shopping activities respectively); leisure is assumed to be performed indifferently at home or at the shopping location. Note that in the experiments, the order of the chain (W and Sh) is irrelevant as the three locations are set on the same straight line.

Before introducing the hierarchical optimization framework, let us write the classical deterministic and instantaneous utility maximization problem, where the individual decide everything at a single instant (what is called macro state in the general modeling approach). All the variables are normalized to cover a single working day. Then, the optimization problem (at one decision level) can be summarized by the following set of equations:

$$\begin{aligned}
 & \underset{i, x_{sh}, T_w, T_{le}, l_w, l_{le}}{\text{Max}} \quad U_n(x_{sh}, T_w, T_{le}, t_v^i) \\
 & \text{s.t.} \\
 & T_w + T_{le} + t_v^i(l_w, l_{le}) = \tau \\
 & p(l_{sh}) \cdot x_{sh} + c_v^i(l_w, l_{sh}) = w(l_w) \cdot T_w \\
 & x_{sh} \geq x_{\min} \\
 & T_{le} \geq \psi \cdot x_{sh}
 \end{aligned} \tag{4}$$

In set of equations (3), the decision variables are the travel mode  $i$ , the aggregated measure of goods  $x_{sh}$ , the work and shopping locations  $\ell_w, \ell_{sh}$ , and the total number of hours of work and leisure  $T_w, T_{le}$ . The problem is written in terms of the direct utility function associated with individual  $n$ , which depends on goods, working and leisure times and on travel time of the chosen mode  $i$ , namely  $t_v^i$ . The two first constraints are the typical time and monetary resource constraints;  $\tau$  is the total of 16 hours available after discounting sleeping hours. The last two constraints are physical constraints; the former is a survival condition establishing that the individual has to consume a minimum amount of goods  $x_{\min}$ , and the latter accounts for a minimum consumption time during the leisure period (defining a minimum consumption rate of  $\psi$ ).

Let us then introduce the multi-level approach associated with problem (4). Basically, we assume that the individual has three levels of decision. At a macro level, s/he can



optimize everything, i.e.  $i, x_{sh}, T_w, T_{le}, \ell_w, \ell_{sh}$ . In a meso level, the individual cannot change the work place, although s/he can modify the assignment of hours to work, i.e., the optimization variables at this level are  $i, x_{sh}, T_w, T_{le}, \ell_{sh}$ . Finally, at the micro level, the individual cannot take decisions regarding the work assignment (neither location not time assignment), therefore s/he optimize over  $i, x_{sh}, T_{le}, \ell_{sh}$ . This multi-scale assumption is quite realistic, as the individual cannot change the workplace every time a shock occurs, unless the shock is alter the long term conditions. The assignment of work hours is also something s/he can decide with less flexibility than the leisure and shopping decisions that are more related to a daily-based decision. This modeling structure will of course provide a more flexible and realistic representation of the real individual behavior than that assumed by the simultaneous single stage model, because we claim that the optimization problem (4) cannot be solved and decided every time a short term shock occurs; alternatively we assume that whole optimization (macro-level) is decided only at certain instants. In the remaining snapshots, the individual can make decisions only of the meso and micro types.

In the next sub-section we utilize this simple framework to show how far from the dynamic hierarchical behavior the modeler can be if always considering the consumers as static single stage optimizers such as (4), which we expect that in most temporal spots should overestimate the real utility that each individual can reach.

### 3.2. Some illustrative results

The total length of the geographical dimension is  $L = 10$  kms. and we model 40 temporal snapshots. We introduce a change in the price of goods (i.e. in the first function in (3)), by modifying  $p_{\min}$  and keeping the other parameters constants. The direct utility function is a Cobb-Douglas in the variables and for these experiments it is defined as follows:

$$\log U = -0.69 - 0.5 \cdot \log(T_w + 1) + 0.4 \cdot \log(x_{sh} + 1) + 0.3 \cdot \log(T_{le} + 1) + \sigma \log(t_v^i)$$

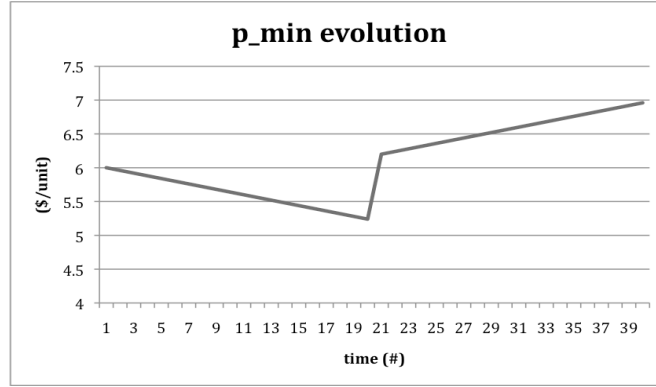
$$\text{where } \sigma = \begin{cases} -0.3 & \text{if } i = \text{bus} \\ -0.1 & \text{if } i = \text{auto} \end{cases}$$

The parameters are consistent with the theory, considering the factors that normally users either like or dislike in their decision behavior. The operational cost of cars is set to 29 (\$/hour) while the bus system is much cheaper (around of 1 \$/trip, fixed). The average speed of cars and buses is set to 30 and 20 kms/hour, respectively. The individual is free to use either auto or bus to accomplish the daily activities.

The parameters for the Gaussian expression in the case of wage rate are  $w_b = 0.1, w_f = 1.5, \ell_w^{\max} = 0.5, w_c = 0.2$  (see expression 3 above). Notice that the maximum salary is obtained in the middle of the total decision stretch.

In what follows, we will test the a single shock produced by the change in the minimum price  $p_{\min}$  over the 40 snapshots as shown in Figure 3, which combines periods of smooth increase and decrease in the minimum prices and a sudden change in this

parameter. The location of the minimum price is also fixed at  $\ell_{sh}^{\min} = 6$  kms. (i.e., the most attractive shopping location is one kilometer beyond the most attractive job location). Hence, this case can capture the trade off between locations as shopping at the lowest price involves extra travel from the optimal job location. The other parameters are set as follows  $p_f = 16, p_c = 14$ .



**Figure 3: Evolution of the minimum goods price over time**

In addition, we consider that the macro and meso decisions are made every 6 and 3 snapshots respectively, and the micro decisions are made at every moment. The solution via simulation is conducted by discretizing both the time and space dimensions (with step sizes of 60 seconds and 100 meters respectively). We sweep over all the feasible space, finding the optimal individual decision according to the constraints associated with either a macro, meso or micro levels. In the next figures, we compare the simulation according to these hierarchical criteria against the theoretical optimal simultaneous –one stage- decisions made at each snapshot (the individual is assumed to make a macro decision at every decision instant).

In the next figures we show the results in terms of utility comparison, time assignment comparison and location of shopping as in these experiments the work location does not change (in all experiments the work location remains the same, at the middle of the stretch where the salary is the highest). These results of course depend on the parameters used, but a fixed work location is both plausible and practical for our analysis because it concentrates the analysis in less variables.  $x_{\min}$  is set at 10 units, and the associated constraints turned out to be always active. Moreover  $\psi = 1000$ , resulting in a constraints of minimum consumption time that is not active, i.e.  $T_{le} > \psi \cdot x_{sh}$ , which is reasonable as the individual always consumes the minimum.

Next, in Figures 4 and 5 we graphically show the most interesting results of our experiments, for the 40 snapshots and comparing the two behavioral models. The one stage macro model (always optimizing everything behind any change in the price market) and the multi-level model, optimizing macro, meso and micro depending on the snapshot as defined above. While in Figure 4 we depict the different time assignments (leisure, work and travel time), in Figure 5 we show the changes in shopping locations (as we said that work location did not change at all) as well as the direct utility levels from the Cobb-Douglas specification evaluated at each snapshot.

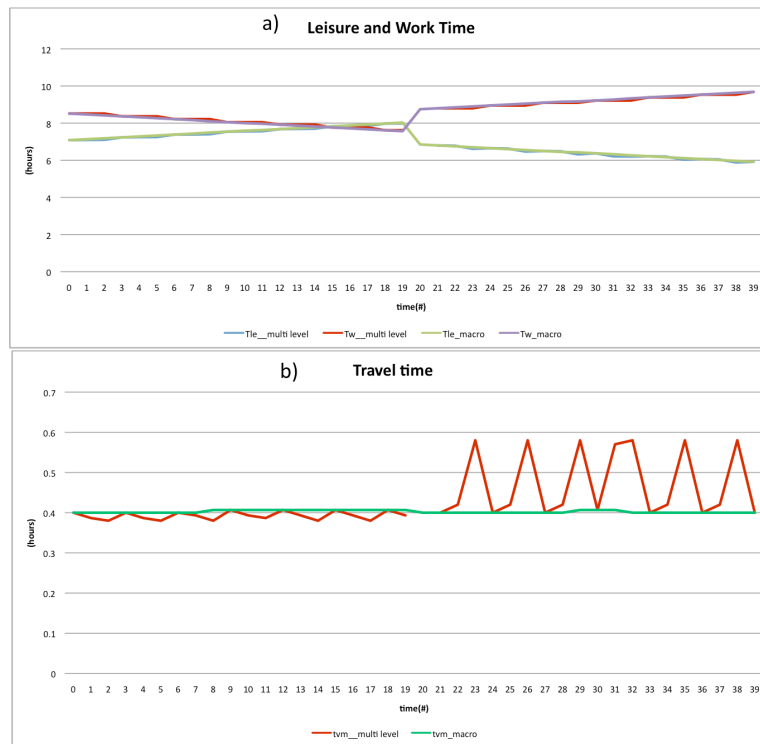


Figure 4: Simulation results: time assignment comparison

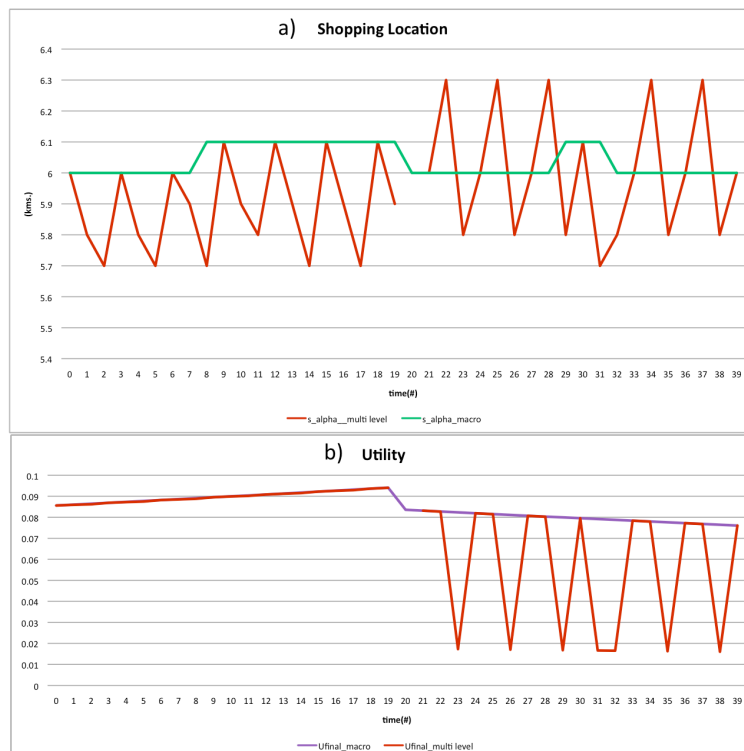


Figure 5: Simulation results: Shopping location and utility

From Figure 4 a), we appreciate that both leisure and work time assignments are similar regardless of the decision, both increase leisure and reduced working time as prices increase. This result, valid for the parameters used, shows that even though sometimes the individual is not able to accommodate everything, indirectly s/he modifies other variables in order to adjust working and shopping time. In or simulation the goods consumption was always very close to the minimum required, showing that the associated utility was not attractive enough to increase consumption..

Figure 4b) shows transport mode choice. The individual initially chooses private car (much more expensive than bus) always in the one stage case, and also initially in the multi stages case and during the time goods prices decrease; however once the goods prices suddenly increase, (from snapshot 22 to 23) the individuals' budget constraint refrain s/he from using car due its high cost and because a minimum goods consumption is required anyway. From that point onwards, mode choice is unstable alternating between car and bus in the hierarchical model. Conversely, despite the sudden jump in prices, the one stage macro option is able to accommodate the resources to allow the individual to keep using car at any moment independent of the market price conditions. Notice that the snapshot where the price suffers a big shock turns out to be unfeasible in case of the multi-level decision, unlike the one stage macro option where feasibility is always reached.

Figure 5 shows the same results in terms of shopping location (not always at the minimum price location) and also in terms of direct utility. The shopping location oscillates around the optimal location while the utility is almost the same for the first period but after the shock it becomes unstable also because of the change of mode in some snapshots mainly as a result of on a strong resource constraint limiting the individual decision.

From this simple example, we can see how different the multi-level assignment can result compared with a one stage macro optimization at any temporal frame, which of course can induce wrong conclusions with regard to the individual's optimal behavior. Notice that if individuals do make long term and short term choices, then their observed behavior can be jumpy or unstable with choice variability as shown in the multi-stage model, while the behavior normally assumed is the one stage macro model of behavior which rather smooth,

## 4. Empirical evidences from Santiago City

### 4.1. Model properties

The theoretically described activity travel model is currently under implementation for Santiago City. The objectives are the development and application of the activity-based approach of travel demand considering the bi-scales hierarchical structure introduced here. Although the focus is on an implementation close to the theoretical framework, data limitations sometimes force us to reduce complexity and simplify the calculation processes. As main input sources for the calculations we use Santiago's Travel and

Household Survey (EOD) and input/output data of Santiago's transport model ESTRAUS and the land use model MUSSA (Sectra, 2002; Florian et al., 1999; Mideplan, 2003; Martínez, 1996).

In general, several steps are addressed during the model setup. The following Figure 6 gives an overview of the model properties and the related data sources we use.

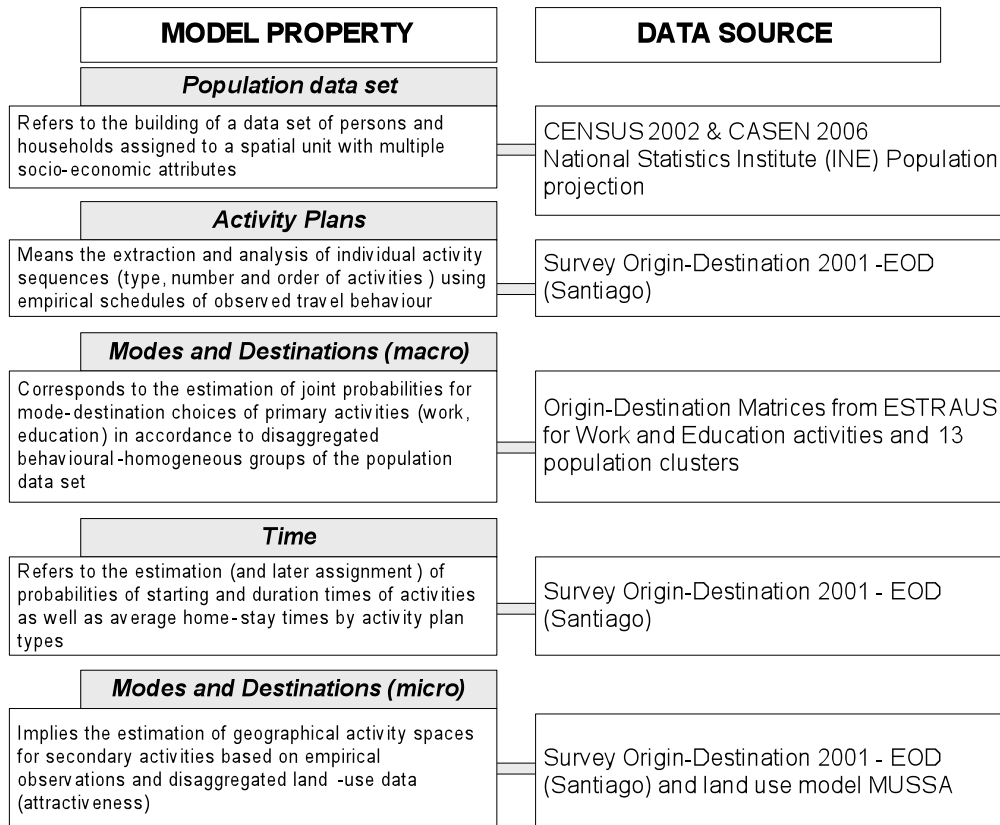


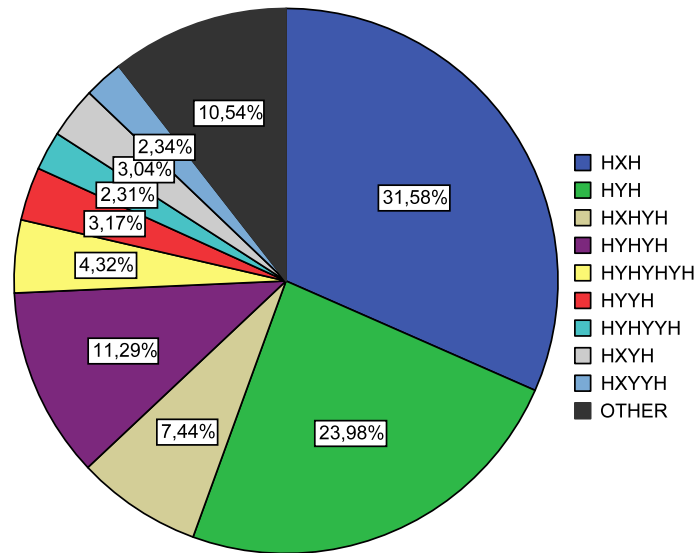
Figure 6: Model properties and data sources

#### 4.2. Activity and Travel Behaviour in Santiago

The population data set is built using standard techniques like the calculation of general statistics (frequencies, cross tables) of the publicly available data sources, as well as regression models, e.g. to estimate car or driving license ownership rates. Other applications showed that the method of Iterative Proportional Fitting (IPF) can be applied to combine aggregated demographic data of persons and households with disaggregated information in form of multi-dimensional cross-tables about socio-economics of persons and households. The IPF procedure serves to adjust iteratively the internal structure of a cross-table to given row sums (Beckman et al., 1996; Hobeika, 2005).

The second step aims at delivering activity plans as described in 2.2 analyzing the empirical data of the Origin-Destination Survey collected in 2001 (EOD-2001). Santiago's Travel and Household Survey provides information about approximately 150,000 trips as well as socio-demographic and economic characteristics of nearly 60,000 people living in 15,000 households. Furthermore the survey includes

geographical coordinates for the work and home-locations and the realized trips. For the analysis we defined work and education activities as primary activities (denoted with 'X') and all other activities of shopping, leisure and other types as secondary activities (denoted with 'Y', the 'H' represents the home location). Considering this we got the following frequency distribution shown in Figure 7 of activity plans for Santiago from the EOD-2001:



Source: EOD-2001, 41.922 activity plans

Annotation: The legend item "OTHER" comprises all activity plan types that occur with a share of < 2%.

**Figure 7: Frequencies of activity plan types in Santiago**

With regard to implications for model building it can be seen that the great majority of activity plans contains one or more simple home-based activities-trips combinations such like going from Home to any primary or secondary activity and going back home (78,6% of all plans). This picture gets even more evident when we analyze not only the complete plan of all activities-trips along the day but compare similar tours among different plans. For example, the plan Home-Work-Home-Shopping-Home consists of two tours starting at home, one going to work and the other going to a shopping facility. If we now add the statistics of tours to the already shown activity plans, we get the following frequency distributions:

**Table 2: Frequencies of Activity Plans and Tours in Santiago (EOD-2001)**

10 most frequent ...				
Plans (complex chains)	Share in %		Tours (home-based)	Share in %
HWH	18,51		HSH	26,84
HEH	13,09		HWH	18,02
HSH	8,39		HLH	15,77
HLH	8,32		HOH	13,35
HOH	7,28		HEH	13,14
HSHSH	3,74		HWSWH	0,94
HSHLH	1,89		HSSH	0,91
HEHLH	1,36		HOOH	0,89
HWWSH	1,25		HOSH	0,69
HOHOH	1,19		HLLH	0,56
Sum	65,0			Sum
N 41.922 Activity Plans			N 59.647 Tours	

With the 10 most frequent plans, 65% of the observed individual travel behaviour in the EOD can already be reproduced. When dividing these plans into tours, 91% of the overall behavioural variance is covered by the top 10 tours. This leads to an average proportion of 1.4 tours per day and person interviewed in the survey. Based on these findings, the conclusion is to concentrate the implementation of the methodology on the plans, respectively tours, identified as the most important. With tours being the basis for the calculation of trip paths - as activities, trips and related decisions are taken within a loop starting and ending at home - the results show that beside the single home-based activities-trips combinations, the appearance of two consecutive out-of-home activities has to be addressed in the implementation. Hence, the following exemplifying remarks refer to an activity plan of the type 'HXYH', representing the conditionality of two out-of-home activities and considering the denotation for 'X' and 'Y' as introduced above.

#### 4.3. Calculation of activities-trips combinations: First Results

For the estimation of mode-destination probabilities of the primary activities of work and education we use the information provided by Santiago's 4-step model ESTRAUS. The model delivers morning-peak OD-matrices disaggregated to 13 user classes (households by income and car-ownership) and for three trip purposes (Work, Education, Other). Meanwhile no better information is available we interpret the trip flow matrices as probability arrays and use them to assign the mode-destination probability for primary activities. Therefore, we calculate the flow sum by every origin-zone and mode to divide then every entry by its row sum. The conversion of the flow matrices is done for all ESTRAUS output matrices and can be exemplified as follows:

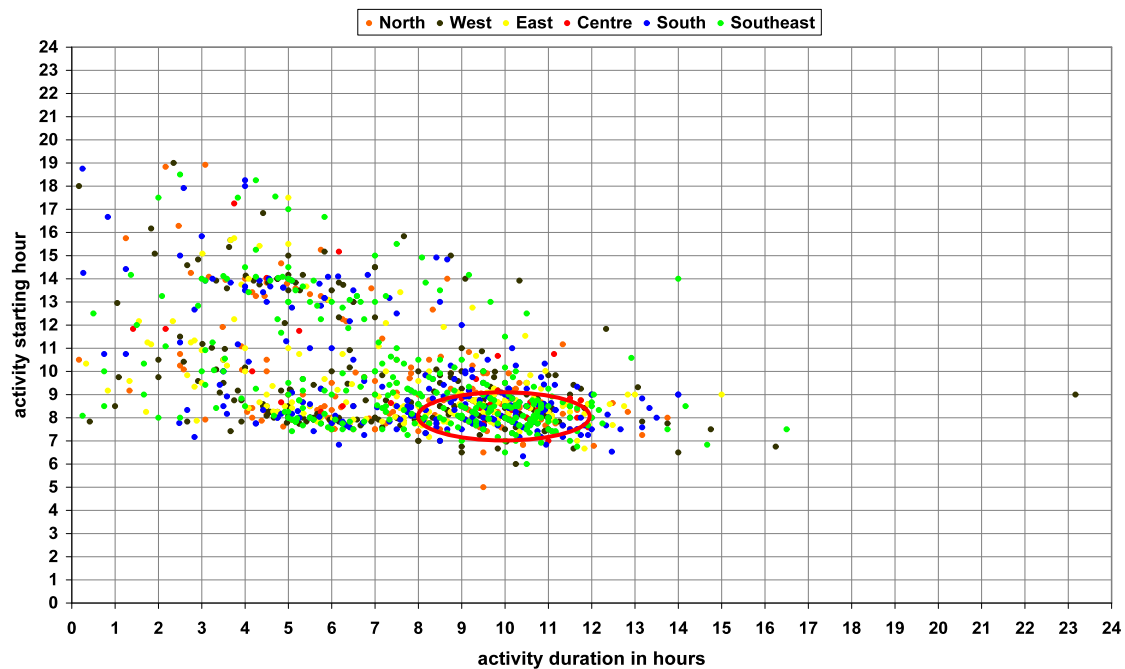
**Table 3: Example for the calculation of mode-destination probabilities**

orig	dest	pcp_06_work	pcd_06_work	pwalk_06_work	ptaxi_06_work	ppt_06_work	pstaxi_06_work
1	1	0	0	0	0	0	0
1	2	0	0	0	0	0	0
1	3	0	0	0	0	0	0
1	4	0,5904	2,452	0,8847	0,0418	0,6939	0,5836
1	5	0,3825	1,6312	1,9075	0,033	0,4335	0,3292
1	6	1,2358	5,0585	1,7135	0,0762	1,4585	0,7691
...	...	...	...	...	...	...	...
Summing up to 1 (or 100%) for every orig (Origin)							

Source: ESTRAS matrices (Model Output)

Annotation: The example refers to a commuting matrix delivered by ESTRAS for the morning-peak hour. The first letter 'p' in the header of column 3 stands for probability, 'cp' for car-passenger, the '06' for user class 6 (households with one car and a monthly income of about 550 to 1.150 US\$). Respectively, 'cd' represents car-driver, 'walk', walking, 'taxi', taxi, 'pt', public transport (bus and metro) and 'staxi', shared taxi. A value of e.g. 5,0585 as in column 4 (car-driver), row 6, denotes the probability that starting from origin-zone 1, a member of user class 06 is going to work in zone 6 by the mode car-driver.

Thus, a great number of the empirically observed simple home-based plans can already be reproduced, e.g. the activity plan Home-Work-Home representing around 18,5% of all plans. So far, temporal aspects have not been considered. Using the empirical data of the EOD we can calculate the distributions of starting and duration times according to activity plan types meanwhile the level-of-service quality indicators (travel times, waiting and transfer times in public transport) are given by the network model that builds part of the transport model ESTRAS. The following Figure 8 shows the information available regarding the timing of the combined primary activities (Work and Education) within the activity plan type 'HXYH'.



Source: EOD-2001, N 1.274 activity plans

**Figure 8: Combination of starting and duration times by Santiago's city sectors**



Annotation: Every position of a point reflects the combination of the starting hour and the duration of the primary activity (both Work and Education are included in the analysis, additionally differentiated by Santiago's city sectors). Both axis display the time in hours, e.g. the points of the highlighted cluster refer to activities that start between 7h and 9h in the morning with a duration of 8 to 12 hours.

The analysis is done based on discrete time hours, which means the probability for any starting and duration time combination that empirically is reported minute-wise, is now summarized to hourly-packages. As expected, the figure allows for the identification of starting and duration time clusters. This is important when calculating conditional probabilities, because not all theoretically feasible starting and duration time combinations have to be considered. In fact, the example indicates a concentration of starting and duration times (7h to 9h; 13h to 14h) and the possibility to exclude certain combinations from the analysis, such as very long activity durations (> 15h), starting after 16h in the afternoon.

In accordance to the theoretical assumptions and staying with the exemplary activity plan of 'HXYH', we now have to condition the starting and duration times of any secondary activity to the distributions assumed for the primary activity. This consideration of a hierarchical decision-making is similar regarding the spatial destination choices for secondary activities that - for the present - are integrated using the OD matrix of the travel purpose 'Other' provided by ESTRAUS. In other words: if the activity plan indicates a working time of 9 hours and an overall out-of-home time of 13 hours, there exist different probabilities that the secondary activity will take 1 to 3 hours. As already said, at this stage of application we work on hourly-based probabilities for the duration and starting times of activities. After the assignment of probabilities for any starting and duration time combination and considering estimated generalized costs (in time) by mode for any feasible mode combination throughout the plan, the overall daily time spent for activities and trips can be calculated. Actually we then check if the overall time spent for each calculated individual program (probabilities for the combination of activity plans + trip paths, see 2.2) remains within the criteria of not exceeding 24h. If this rule is broken, the respective programs probabilities are set to 0 while the share of the feasible programs is re-weighted.

The consideration of a very huge number of behavioural combinations (starting and duration times, mode-destination combinations) tends to produce small probabilities, thus leading to marginal values for the activities-trips flows. In the following Table 4 underlying probabilities for complete activities-trips combinations for the activity plan type 'HXYH' were multiplied with a population of 15.000 persons living in zone 205. The first four columns indicate the macro zones of the trip paths combinations, e.g. with the home-location in zone 205 travelling to zone 48 for work, then to zone 7 for shopping and back to 205. Similar to Table 3 the underlying probabilities sum up to 1 (or 100%) by each origin, but this time they are more differentiated as two destinations ('X' and 'Y') and all possible mode-destination and time combinations are considered. Respectively, Table 4 shows only a part of this large table, concentrating on trips starting between 7h and 8h in the morning, lasting until a point in time between 17h and 18h followed by a shopping activity of one hour finishing the sequence between 18h and 19h using exclusively public transport modes throughout the plan. The results

show that in many cases the activities-trips flows remain below 0, hence representing only a partial flow.

**Table 4: Exemplary output of activities-trips flows for the activity plan type 'HXYH'**

ORIGIN	DESTINATION	DESTINATION	ORIGIN	dem 7 17 18	dem 7 17 19	dem 7 17 20	dem 7 17 21	dem 7 19 20
205	48	7	205	,667	,600	,107	,007	,107
205	48	8	205	1,187	1,068	,190	,012	,190
205	48	18	205	,837	,753	,134	,008	,134
205	48	20	205	2,566	2,309	,411	,026	,411
205	48	23	205	2,349	2,114	,376	,023	,376
205	48	28	205	2,516	2,265	,403	,025	,403
205	48	32	205	,923	,830	,148	,009	,148
205	155	7	205	,574	,517	,092	,006	,092
205	155	8	205	,947	,852	,151	,009	,151
205	155	28	205	3,645	,648	,041	,041	,041
205	155	45	205	,852	,767	,136	,009	,136
205	155	46	205	1,350	1,215	,216	,014	,216
205	155	47	205	1,486	1,337	,238	,015	,238
205	157	50	205	,322	,290	,051	,003	,051
205	157	51	205	,448	,403	,072	,004	,072
205	157	56	205	3,042	2,738	,487	,030	,487
205	157	64	205	3,789	,674	,042	,042	,042
205	157	65	205	2,639	,469	,029	,029	,029
205	157	72	205	1,082	,974	,173	,011	,173
205	157	78	205	,462	,416	,074	,005	,074
205	157	80	205	1,597	1,437	,256	,016	,256
205	157	116	205	3,946	,702	,044	,044	,044

Although it can be shown that the calculations are technically feasible based on the empirical observations of the EOD and the model output delivered by ESTRASUS, the fragmentation of the transport demand seems too disaggregated. Nevertheless the analysis on time dependencies as well as the introduction of a geographically restricted destination choice can reduce the combinatorial number, thus increase the probabilities for activities-trips combinations.

## 5. Conclusions and Outlook

Regarding the attempts of bringing the theoretical framework into practice the results achieved so far are promising. The methodology and its main property of hierarchical decision-making in time and space are implemented in a first version of the model working fully probabilistic. Nevertheless further efforts facing the combinatorial problem and the deepening of the behavioral and statistical analysis are necessary. With regard to time aspects, the relevant dependencies between starting and duration times have to be identified against the background of relevant variables that determine the timing of plans, e.g. socio-economic or spatial attributes. Additionally, the relevant time-dependent clusters have to be identified to reduce the amount of considered combinations. Addressing the assignation of activity plans to members of the population, the relevant person and household attributes need to be ascertained that mark differences in activity behavior.

In addition, we tested the multi-level framework through simulation in Section 3, checking by means of a simple deterministic example the premise that justify this modeling approach, showing that the individual is not always able to react in a short temporal frame to market shocks that eventually in our experiments can bring the solution towards unfeasible points. We compare the realistic behavior against a

benchmark where everything can be optimized at any snapshot. Results show that shocks (in this case directly associated with the price market) can provoke serious bias in the computation of utility functions when a static mono-level approach is used instead of recognizing the different scales in the individual decision process.

To deal with the combinatorial problem the so far not treated issue of determining geographical activity spaces will reduce the set of options reasonably. For that purpose the EOD is analyzed with respect to geographical destination choices and travelled distances. Thus, for complex activity plans with dependent activity and trip choices of the type 'HXYH' the number of spatial options decreases, reducing possible combinations and calculation efforts.

The next steps aim at the implementation of the fully probabilistic framework for the calculation of activities-trips combinations, including their differentiation in time and considering manifold types of activity plans. Results in the form of interconnected OD-matrices as shown in Table 4 may then be split once again into individual pair-wise OD matrices and assigned to transport networks. The altered level-of-service indicators are then recognized once again in the process of demand generation and the calculation of the activities-trips combinations. Doing so, variances in supply-quality during the day may be considered making the approach realistic to actual conditions of the transport system.

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## 6. References

- Beckman, R. J., Baggerly, K. A. and McKay, M. D. (1996): Creating Synthetic Baseline Populations. *Transportation Research Part A* 30(6), pp. 415-429.
- Bhat, C. et al. (2008): CEMDAP: Modelling and Microsimulation Frameworks, Software Development and Verification. *Transportation Research Board Paper DVD*, pp. 1-23. Washington.
- Bowman, J. L. and Ben-Akiva, M. E. (2000): Activity-Based disaggregate travel demand model system with activity schedules. *Transportation Research Part A* 35(1), pp. 1-28.
- Buliung, R. N. and Kanaroglou, P. S. (2007): Activity-Travel Behaviour Research: Conceptual Issues, State of the Art, and Emerging Perspectives on Behavioural Analysis and Simulation Modelling. *Transport Reviews* 27(2), pp. 151-187.

De Serpa, A. (1971): A Theory of the Economics of Time. *The Economic Journal* 1, pp. 828-846.

Florian, M., Wu, J. H. and He, S. (1999): A multi-class multi-mode variable demand network equilibrium model with hierarchical logit structures. Montreal, Canada.

Gunderson, L. H. and Holling, C. S. (2002): Panarchy: Understanding transformations in human and natural systems. Washington.

Hobeika, A. (2005): Population Synthesizer. TRANSIMS Fundamentals, Chapter 3, pp. 1-117.

Hunt, J. D. and Abraham, J. D. (2007): PECAS for Spatial Economic Modelling, Theoretical Formulation, Working Draft, Calgary, Alberta.

Iacono, M., Levinson, D. and El-Geneidy, A. (2008): Models of Transportation and Land Use Change: A Guide to the Territory. *Journal of Planning Literature* 22(4), pp. 323-340.

Justen, A. and Cyganski, R. (2008): Decision-making by microscopic demand modelling: a case study. Paper presented at the International Conference on Transportation decision making: issues, tools, models and case studies. Venice, Italy.

Martínez, F. (1996): MUSSA - Land Use Model for Santiago City. *Transportation Research Record* 1552, pp. 126-134.

Ministerio de Planificación - Mideplan (2003): ESTRAUS. Manual del Usuario. Versión 4.3. Santiago, Chile.

Rilett, L. R. (2001): Transportation Planning and TRANSIMS Microsimulation Model. Preparing for the Transition. *Transportation Research Record* 1777, pp. 84-92.

Secretaría Interministerial de Planificación de Transporte - Sectra (2002): Mobility Survey EOD 2001. Executive Report. Santiago, Chile.

Davidson, W; R. Donnelly, P. Vovsha, J. Freedman, S Ruegg, J. Hicks, J. Castiglione and R. Picado. (2007): Synthesis of first practices and operational research approaches in activity-based travel demand modelling. *Transportation Research Part A* 41(5), pp. 464-488.

Waddell, P. (2002): URBANSIM: Modelling urban development for land use, transportation, and environmental planning. *Journal of the American Planning Association* 68(3), pp. 297-314.